8 SLICOT Controls Microwaves – a Case Study

Motivation

Recent technological advances in microwave household appliances have created the possibility of a more refined control of the heating process in combination ovens. This includes the control of both a microwave heating source and a forced convection (air) heating source. Optimal control of a combination oven will give a higher quality end-product with a more uniform temperature distribution and, hence, a better cook-quality. This short article reports on the application of the (recently available) SLICOT reduction routines for calculation of an optimal control history for a microwave combination oven. We present a simple, finite horizon linear quadratic regulator solution to the heating problem, which was calculated using the reduced set of model equations. The example involves the calculation of an optimal heating profile for a container of mashed potato. The original (high dimensional) system model was reduced using the SLICOT routine AB09CD, which includes optimal Hankel norm approximation with square-root balancing. This routine allows a substantial reduction of computation time for the LQ design. The reduction routines were found to be more efficient than MATLAB routines. Recent progress is reported.

Fourier's Law of Heat Diffusion

A simplified model of heat conduction can be described by Fourier's heat equation which, in Cartesian coordinates $\underline{r} = (x, y, z)$, reads

$$\rho(\underline{r})c(\underline{r})\frac{\partial T(\underline{r},t)}{\partial t} = \vec{\nabla}\{k(\underline{r}) \cdot \vec{\nabla}T(\underline{r},t)\} + Q(\underline{r},t) \tag{3}$$

where $T(\underline{r},t)$ is the temperature field (^oC), $Q(\underline{r},t)$ is the heat flux which drives the system to a desired state or final temperature field, $\rho(\underline{r})$ is the density of the body to be heated, and $c(\underline{r})$ is the heat capacity. We note that in reality heat conductance and heat conductivity are temperature dependent and, hence, the above model is only a good approximation of the heat diffusion process on a restricted temperature domain. Further, note that the heat flux $Q(\underline{r},t)$ in the heat diffusion PDE consists of two heating sources, namely a convection boundary condition for nodal points $r_i \in \partial\Omega$ and internal heating through microwaves for the points $r_i \in \Omega$. For boundary points the surface boundary condition expresses the heat transfer from the air to the object, i.e.

$$\forall \underline{r} \in \partial\Omega : \quad -k(\underline{r})\frac{\partial T(\underline{r},t)}{\partial n} = h_q(T(\underline{r},t) - T_\infty(t)) \tag{4}$$

where h_q is the surface heat transfer coefficient, n is the (outwards orientated) normal vector to the surface $\partial\Omega$, and $T_\infty(t)$ is the air temperature in the oven cavity.

The temperature field $T(\underline{r},t)$ is approximated using finite element (Galerkin) approximation techniqes which transform the system model to a set of ordinary differential equations. Assuming spatial uniformity of densities, heat capacities, and conductivities, this gives

$$C_p\frac{dx(t)}{dt} + Kx(t) = f(\underline{r})\begin{pmatrix} P(t) \\ T_\infty(t) \end{pmatrix} \tag{5}$$

with C_p the heat capacitance matrix $(N \times N)$, K the conductivity matrix $(N \times N)$, $f(\underline{r})$ the heat input vector $(N \times m)$, and $P(t)$ and $T_\infty(t)$ are the μ-wave power field and the air temperature in the oven cavity respectively. The N nodal temperatures $x_i(t)$ are defined on a grid κ. The dimension of the input vector (m) equals two in this case. Note that the driving term on the right hand side of (5) has been separated into a spatial dependent and a temporal dependent component.

SLICOT Reduction

The above set of ordinary differential equations in the *nodal temperatures* $x(t)$ on a grid κ is, in fact, the starting point for the control analysis. The dimension N of the nodal temperature vector can be quite large (order of magnitude is several thousands of nodes for a realistic case study) and the system matrices C_p and K are *sparse*. Calculation of the optimal, finite horizon, linear quadratic regulator for the full system is very laborious and involves too many equations. Hence, SLICOT's routine AB09CD was used to reduce the original set of equations (5) to a more workable size, i.e.

$$\dot{x}_r(t) = A_r x_r(t) + B_r u(t) \quad (6)$$

$$y(t_k) = C_r x_r(t_k) \quad (7)$$

with $\dim(x_r) << \dim(x)$. It was found that the efficiency of the AB09CD routine outperforms similar reduction commands in Matlab, especially for large dimensional problems with several thousands of nodes. A comparison (expressed in CPU time) was made between a reduction using Matlab's command 'ohklmr' and the SLICOT routine AB09CD. It was found that, for this particular problem, SLICOT performed 18 times faster (41.5 seconds, instead of 750.3 seconds for Matlab's ohklmr). Also, the memory usage for the mashed potato problem (see below) was approximately a fourth of the size Matlab used to solve the same problem. In order to find an optimal control history, the original quadratic cost function in the nodal temperatures $x(t)$ was transformed to a cost in the reduced state vector x_r and the associated algebraic Riccati equation in the reduced state was simply swept back to t_0.

An Example Case Study

As an example case study the optimal profiles for a cylindrical container (figure 3) of mashed potato will be presented. First, the matrices C_p, K, and the input heating vector $f(\underline{r})$ were calculated with Galerkin finite element approximation techniques on the basis of the physical properties of mashed potato[5]. The microwave power distribution was approximated using Lambert's law which states that the attenuation of microwave power is exponentially decreasing in space (see figure 4 for the microwave distribution on the surface Ω_1) with an attenuation factor β. The Lambert approximation is not a very good one for realistic case studies but it demonstrates the method used here. We are currently working on examples which include more realistic values for the microwave power distribution, based on (finite difference) simulations of Maxwell's electrodynamical laws in three dimensions.

The original system for the mashed potato problem involves 441 ordinary differential equations. SLICOT's AB09CD routine reduced this set to 41 states which substantially reduced the computations involved in a backward sweep of the algebraic Riccati equations for

[5]These matrices will be available on the SLICOT website.

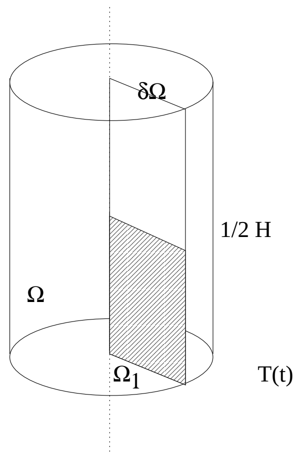

Figure 3: Cylindrical container of mashed potato. The simulations in this study involve the half vertical squared surface Ω_1.

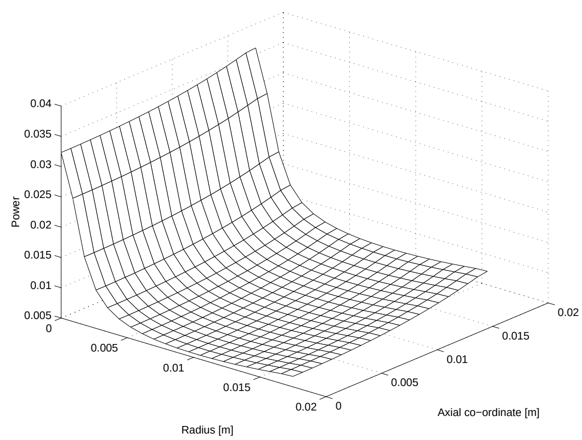

Figure 4: Microwave Power Distribution following Lambert's attenuation law.

calculation of the optimal control history $\{u(t), 0 < t < t_f\}$. It was found that the calculation of an optimal control history using SLICOT's reduced system model took approximately 2000 times less CPU time when compared to a similar calculation for the full 441 dimensional model. This, of course, is solely due to the model reduction. The calculated control inputs are shown in figure 5. Note that, for this specific case study, the microwave power controls the heating process almost completely. The fairly rapid decrease just before the final time instant $t_f = 90s$ is to allow heat diffusion from the center of the cylinder to the boundaries (figure 6).

Although this may not seem plausible on the basis of Lambert's law, the power distribution of the microwaves *decreases* from $r = 0$ to $r = R$ since the circular boundary introduces a focussing effect of microwave power in the center of the cylinder causing a rapid increase in temperature in the center. Indeed, this example demonstrates that the geometry of the problem contributes substantially to the heating process. An interesting additional exercise

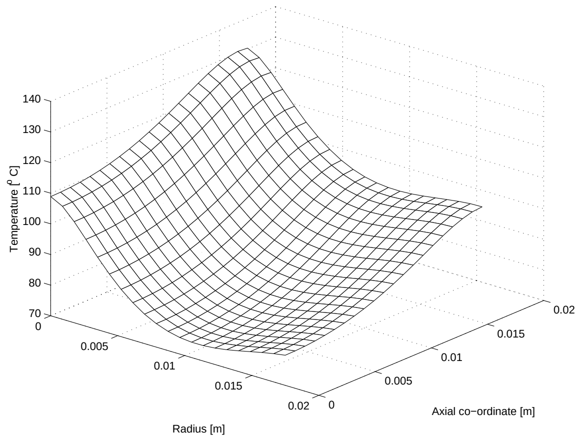

Figure 5: Final temperatures on the surface Ω_1 after 90 seconds of optimally combined hot-air/microwave heating.

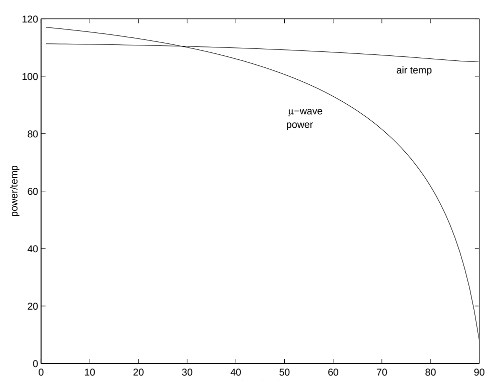

Figure 6: Optimal input profiles for combined μ-wave/forced convection heating for the mashed potato example.

is to calculate an optimal heating profile for convective control only. This was performed for the mashed potato example using a heating time of approximately 25 minutes. For this case SLICOT reduced the system to 29 states. The resulting input heating profile shows a ΔT-cooking profile, meaning that initially the heating temperature needs to be increased to maintain a constant gradient between the center and the boundary of the potato (figure 7) in order to transfer the boundary heat input most efficiently. The oscillation after the initial ΔT profile is to 'fine-tune' the temperature to reach exactly a uniform final temperature field of 100 ^oC. In figure 8 one can see that uniformity is reached within a range of approximately 0.1^oC which is much better than the case of combined μ-wave/forced convection heating.

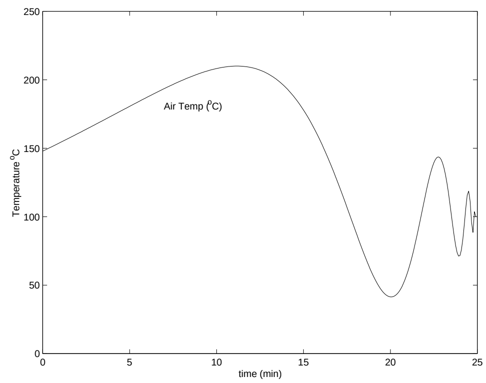

Figure 7: Optimal input profiles for convection heating only.

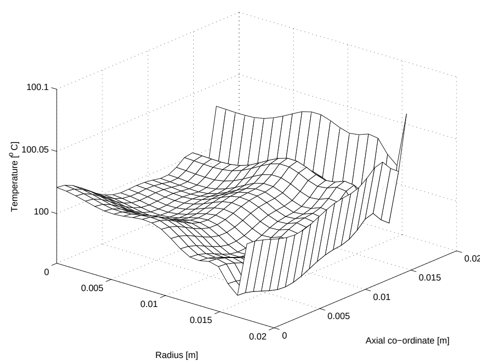

Figure 8: Final temperature distribution for convection heating only.

Closing Remarks

This short note has demonstrated an interesting application of the SLICOT library for finding optimal heating strategies for multimode combination ovens (air/microwave oven). Our experience has shown that the SLICOT routines are very efficient and fast solvers for model reduction and are a useful tool in this application. Future work will involve the application of the same routines to systems with a dimension of approximately 4000 nodes (a test case with a 1500 node model has already been reduced to 61 states in approximately 1 hour of CPU time which is very good).

Acknowledgements

This work was supported by the European Committee (FAIR CT96-1192), and by the Flemish Community under contract number COF-97-08.

References

[1] A.E. Bryson, Jr. *Dynamic Optimization.* Addison Wesley, 1999.

[2] A.E. Bryson, Jr. and Y. Ho. *Applied Optimal Control: Optimization, Estimation, and Control.* John Wiley & Sons, 1975.

[3] K. Åström and B. Wittenmark. *Computer Controlled Systems: Theory and Design.* Prentice-Hall, 1984.

[4] D. Chen, R. Sing, K. Haghighi, and P. Nelson. Finite element analysis of temperature distribution in microwaved cylindrical potato tissue. *Journal of Food Engineering*, 18:351–368, 1993.

Hans Stigter, Nico Scheerlinck, Bart Nicolaï, and Jan van Impe